Practice Questions for Validation Exam for MA1114

No calculators allowed.

Note: The questions below indicate the types of questions on the actual exam. For each topic shown, the actual exam has one problem similar to the one of the questions in the list.

A. **Applications of Integrals**

1. Find the area *enclosed by* the two curves

$$x = \tfrac{3}{8}\, y^3 \qquad \text{and} \qquad y = -\tfrac{2}{9}\, x^2$$

2. Find the area *between* the two curves

$$y = 2\,\cos(x/2) \qquad \text{and} \qquad y = \sin^2(x)$$

for $0 \le x \le \pi$.

3. Find the area *enclosed by* the three curves

$$x = 0 \qquad \text{and} \qquad y = e^x - 1 \qquad \text{and} \qquad y = 2\,e^{-x}$$

B. **Sequences and Series**

1. Given the definition below for a_n

$$a_n = \frac{n^3}{2^n}$$

   (a) Does the *sequence* $\{a_n\}$ converge? if so, to what?

   (b) Does the *series* $\sum_{n=1}^{\infty} a_n$ converge? Justify your answer with an appropriate test.

2. Given the definition below for a_n

$$a_n = \frac{\sqrt{n+1}}{n+5}$$

   (a) Does the *sequence* $\{a_n\}$ converge? if so, to what?

   (b) Does the *series* $\sum_{n=1}^{\infty} a_n$ converge? Justify your answer with an appropriate test.

3. Given the definition below for a_n

$$a_n = \frac{n^2 + 2^{-n}}{1 + 4n - 3n^2}$$

   (a) Does the *sequence* $\{a_n\}$ converge? if so, to what?

   (b) Does the *series* $\sum_{n=1}^{\infty} a_n$ converge? Justify your answer with an appropriate test.

C. **Taylor Series and Power Series**

1. Given the Power Series below:

$$\sum_{n=0}^{\infty} \frac{(1 - 2x)^n}{\sqrt{2n+1}}$$

   (a) For what values of x does the series *converge absolutely*?
   (b) For what values of x does the series *converge conditionally*?
   (c) For what values of x does the series *diverge*?

2. Find the Taylor Series for the function $f(x) = \sin x$ about the point $x_0 = \pi/4$.

3. Find an *approximate* solution x to the equation

$$\cos x = x$$

   using the second-order Taylor polynomial $P_2(x)$ (about $a = 0$) to approximate $\cos x$. Note: the equation has only one solution. Estimate the accuracy of your approximate solution (find a bound on the error).

D. **Systems of Linear Equations**

1. Find the solution set of the system of equations below.

$$\begin{array}{rcrcrcl} -x_1 & + & x_2 & + & 2\,x_3 & = & 3 \\ 2\,x_1 & & & + & x_3 & = & 4 \\ x_1 & + & x_2 & + & 3\,x_3 & = & 6 \end{array}$$

2. Find the solution set of the system of equations below.

$$\begin{array}{rcrcrcl} 2\,x_1 & - & x_2 & + & x_3 & = & 0 \\ -x_1 & + & 2\,x_2 & + & 3\,x_3 & = & -2 \\ 3\,x_1 & & & + & 5\,x_3 & = & -2 \end{array}$$

3. Find the solution set of the system of equations below.

$$\begin{array}{rcrcrcl} x_1 & - & 2\,x_2 & + & 2\,x_3 & = & 0 \\ 2\,x_1 & + & x_2 & + & x_3 & = & -3 \\ x_1 & & & + & 3\,x_3 & = & 1 \end{array}$$

E. **Matrix Eigenvalues and Eigenvectors**

1. Find the eigenvalues and eigenvectors of the matrix A below.

$$A = \begin{pmatrix} -1 & -2 \\ -2 & 2 \end{pmatrix}$$

2. For the matrix A below, find the eigenvectors that correspond to the eigenvalue $\lambda = 1$.

$$A = \begin{pmatrix} 3 & 1 & -1 \\ -4 & -1 & 2 \\ -2 & -1 & 2 \end{pmatrix}, \quad \lambda = 1$$

3. Find the eigenvalues and eigenvectors of the matrix A below.

$$A = \begin{pmatrix} -3 & -5 \\ 1 & -5 \end{pmatrix}$$